Formulaire de trigonométrie de terminale C, année 1974-75
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Fascicule 3
(voir fascicule 4)

$\cos(-x) = \cos x$ $\quad \cos(\pi - x) = -\cos x$ $\quad \cos(\pi + x) = -\cos x$

$\sin(-x) = -\sin x$ $\quad \sin(\pi - x) = \sin x$ $\quad \sin(\pi + x) = -\sin x$

$\operatorname{tg}(-x) = -\operatorname{tg} x$ $\quad \operatorname{tg}(\pi - x) = -\operatorname{tg} x$ $\quad \operatorname{tg}(\pi + x) = \operatorname{tg} x$

$\operatorname{cotg}(-x) = -\operatorname{cotg} x$ $\quad \operatorname{cotg}(\pi - x) = -\operatorname{cotg} x$ $\quad \operatorname{cotg}(\pi + x) = \operatorname{cotg} x$

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$ $\quad \cos\left(\frac{\pi}{2} + x\right) = -\sin x$

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$ $\quad \sin\left(\frac{\pi}{2} + x\right) = \cos x$

$\operatorname{tg}\left(\frac{\pi}{2} - x\right) = \operatorname{cotg} x$ $\quad \operatorname{tg}\left(\frac{\pi}{2} + x\right) = -\operatorname{cotg} x$

$\operatorname{cotg}\left(\frac{\pi}{2} - x\right) = \operatorname{tg} x$ $\quad \operatorname{cotg}\left(\frac{\pi}{2} + x\right) = -\operatorname{tg} x$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\sin(a+b) = \sin a \cos b + \sin b \cos a$

$\sin(a-b) = \sin a \cos b - \sin b \cos a$

$\operatorname{tg}(a+b) = \dfrac{\operatorname{tg} a + \operatorname{tg} b}{1 - \operatorname{tg} a \operatorname{tg} b}$

$\operatorname{tg}(a-b) = \dfrac{\operatorname{tg} a - \operatorname{tg} b}{1 + \operatorname{tg} a \operatorname{tg} b}$

$\cos 2x = \cos^2 x - \sin^2 x$ — $\cos^2 x = \dfrac{1 + \cos 2x}{2}$

$\sin 2x = 2 \sin x \cos x$ $\quad \sin^2 x = \dfrac{1 - \cos 2x}{2}$

$\cos 2x = \dfrac{1 - \operatorname{tg}^2 x}{1 + \operatorname{tg}^2 x}$ $\quad \cos x = \dfrac{1 - t^2}{1 + t^2}$

$\sin 2x = \dfrac{2 \operatorname{tg} x}{1 + \operatorname{tg}^2 x}$ $\quad$ ou : $\sin x = \dfrac{2t}{1 + t^2}$ $\quad$ avec $t = \operatorname{tg} \dfrac{x}{2}$

$\operatorname{tg} 2x = \dfrac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$ $\quad \operatorname{tg} x = \dfrac{2t}{1 - t^2}$

Transformation produit - somme.

$\cos a . \cos b = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$

$\sin a . \sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$

$\sin a . \cos b = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$

Transformation somme - produit.

$\cos p + \cos q = 2 \cos \frac{p+q}{2} . \cos \frac{p-q}{2}$ $\quad \sin p + \sin q = 2 \sin \frac{p+q}{2} . \cos \frac{p-q}{2}$

$\cos p - \cos q = -2 \sin \frac{p+q}{2} . \sin \frac{p-q}{2}$ $\quad \sin p - \sin q = 2 \sin \frac{p-q}{2} . \cos \frac{p+q}{2}$

Relations entre $\sin x, \cos x, \operatorname{tg} x, \operatorname{cotg} x$.

$\cos^2 x + \sin^2 x = 1$ $\quad \operatorname{tg} x = \dfrac{\sin x}{\cos x}$ $\quad \operatorname{cotg} = \dfrac{1}{\operatorname{tg} x}$

$1 + \operatorname{tg}^2 x = \dfrac{1}{\cos^2 x}$

$1 + \operatorname{cotg}^2 x = \dfrac{1}{\sin^2 x}$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\operatorname{tg} x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	
$\operatorname{cotg} x$		$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

$\text{Cos angle}(\vec{u}, \vec{v}) = \dfrac{\vec{u} . \vec{v}}{\|\vec{u}\| . \|\vec{v}\|}$

$\text{Sin angle}(\vec{u}, \vec{v}) = \dfrac{\det(\vec{u}, \vec{v})}{\|\vec{u}\| . \|\vec{v}\|}$

* Equations trigo. élémentaires.

$\sin a = \sin b \Leftrightarrow \begin{cases} b = a + 2k\pi \\ b = \pi - a + 2k\pi \end{cases}$ conséquence $\sin x = a$

$\cos a = \cos b \Leftrightarrow \begin{cases} b = a + 2k\pi \\ b = -a + 2k\pi \end{cases}$ conséquence $\cos x = a$

$\operatorname{tg} a = \operatorname{tg} b \Leftrightarrow b = a + k\pi$ conséquence $\operatorname{tg} x = a$

$\operatorname{cotg} a = \operatorname{cotg} b \Leftrightarrow \operatorname{tg} a = \operatorname{tg} b \Leftrightarrow b = a + k\pi$ conséquence $\operatorname{cotg} x = a$

* Equation du type (E): $a \cos x + b \sin x = c$

1 méthode: trigonométrique. 2 méthode: algèbre. 3 méthode: interprétation graphique.

* Dérivation des fonctions circulaires:

$(\sin x)' = \cos x$; $(\cos x)' = -\sin x$; $(\operatorname{tg} x)' = \dfrac{1}{\cos^2 x}$; $\left[(\operatorname{cotg} x)' = -\dfrac{1}{\sin^2 x}\right]$

$\left[\cos(ax+b)\right]' = -a.\sin(ax+b)$; $\left[\sin(ax+b)\right]' = a.\cos(ax+b)$

Formulaire de trigonométrie année 1975

1 Lignes trigonométriques des angles remarquables

Note : tg x est l'ancienne notation de $\tan x$ *(influence ango-saxonne)*

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\operatorname{tg} x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞
$\operatorname{cotg} x$	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

2 Formules élémentaires

$$\sin^2 x + \cos^2 x = 1, \qquad \operatorname{tg} x = \frac{\sin x}{\cos x}, \qquad \operatorname{cotg} x = \frac{1}{\operatorname{tg} x}$$

$$1 + \operatorname{tg}^2 x = \frac{1}{\cos^2 x}, \qquad 1 + \operatorname{cotg} x = \frac{1}{\sin x}$$

3 Formules de symétrie et de déphasage

$$\cos(-x) = \cos x$$
$$\cos(\pi - x) = -\cos x$$
$$\cos(\pi + x) = -\cos x$$
$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$
$$\cos\left(\frac{\pi}{2} + x\right) = -\sin x$$

$$\sin(-x) = -\sin x$$
$$\sin(\pi - x) = \sin x$$
$$\sin(\pi + x) = -\sin x$$
$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$
$$\sin\left(\frac{\pi}{2} + x\right) = \cos x$$

$$\operatorname{tg}(-x) = -\operatorname{tg} x$$
$$\operatorname{tg}(\pi - x) = -\operatorname{tg} x$$
$$\operatorname{tg}(\pi + x) = \operatorname{tg} x$$
$$\operatorname{tg}\left(\frac{\pi}{2} - x\right) = \operatorname{cotg} x$$
$$\operatorname{tg}\left(\frac{\pi}{2} + x\right) = -\operatorname{cotg} x$$

$$\operatorname{cotg}(-x) = -\operatorname{cotg} x$$
$$\operatorname{cotg}(\pi - x) = -\operatorname{cotg} x$$
$$\operatorname{cotg}(\pi + x) = \operatorname{cotg} x$$
$$\operatorname{cotg}\left(\frac{\pi}{2} - x\right) = \operatorname{tg} x$$
$$\operatorname{cotg}\left(\frac{\pi}{2} + x\right) = -\operatorname{tg} x$$

4 Formules d'addition

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$
$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\operatorname{tg}(a+b) = \frac{\operatorname{tg} a + \operatorname{tg} b}{1 - \operatorname{tg} a \cdot \operatorname{tg} b}$$

$$\operatorname{tg}(a-b) = \frac{\operatorname{tg} a - \operatorname{tg} b}{1 - \operatorname{tg} a \cdot \operatorname{tg} b}$$

5 Formules de duplication et de linéarisation

$$\begin{aligned}\cos 2x &= cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \\ \sin 2x &= 2\sin x \cos x\end{aligned}$$

$$\cos^2 a = \frac{1 + \cos 2a}{2}$$

$$\sin^2 a = \frac{1 - \cos 2a}{2}$$

6 Lignes trigonométriques en fonction de l'angle moitié

On pose : $t = \operatorname{tg}\left(\frac{x}{2}\right)$

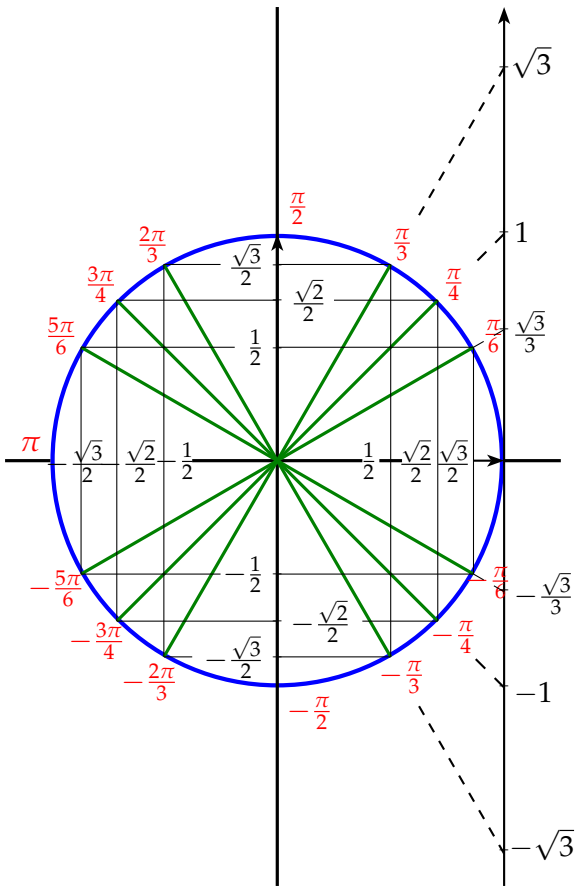
$$\cos x = \frac{1-t}{1+t^2} \qquad \sin x = \frac{2t}{1+t^2} \qquad \operatorname{tg} x = \frac{1-t^2}{1+t^2}$$

7 Transformation produit-somme et somme-produit

$$\cos a \cos b = \frac{1}{2}\left[\cos(a-b)+\cos(a+b)\right]$$
$$\sin a \sin b = \frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right]$$
$$\sin a \cos b = \frac{1}{2}\left[\sin(a-b)+\sin(a+b)\right]$$

$$\cos p + \cos q = 2\cos\frac{p+q}{2}\times\cos\frac{p-q}{2}$$
$$\cos p - \cos q = -2\sin\frac{p+q}{2}\times\sin\frac{p-q}{2}$$
$$\sin p + \sin q = 2\sin\frac{p+q}{2}\times\cos\frac{p-q}{2}$$
$$\sin p - \sin q = 2\sin\frac{p-q}{2}\times\cos\frac{p+q}{2}$$

8 Cercle trigonométrique



9 Équations trigonométriques

- Équations élémentaires

$$\sin a = \sin b \quad \Leftrightarrow \quad \begin{cases} b = a + 2k\pi \text{ ou} \\ b = \pi - a + 2k\pi \end{cases} \qquad \text{conséquence} \quad \sin x = a$$

$$\cos a = \cos b \quad \Leftrightarrow \quad \begin{cases} b = a + 2k\pi \text{ ou} \\ b = -a + 2k\pi \end{cases} \qquad \text{conséquence} \quad \cos x = a$$

$$\operatorname{tg} a = \operatorname{tg} b \quad \Leftrightarrow \quad b = a + k\pi \qquad \text{conséquence} \quad \operatorname{tg} x = a$$

$$\operatorname{cotg} a = \operatorname{cotg} b \quad \Leftrightarrow \quad b = a + k\pi \qquad \text{conséquence} \quad \operatorname{cotg} x = a$$

- Équations du type $a\cos x + b\sin y = c$

  Méthodes : 1) trigonométrique 2) algébrique 3) interprétation graphiques

10 Dérivation des fonction circulaires

$$\sin' x = \cos x \quad , \quad \cos' x = -\sin x \quad , \quad \operatorname{tg}' x = \frac{1}{\cos^2 x} \quad , \quad \operatorname{cotg}' x = -\frac{1}{\sin^2 x}$$

$$\sin'(ax+b) = a\cos(ax+b) \quad , \quad \cos'(ax+b) = -a\sin(ax+b)$$